Grade Level/Course:
Algebra I

Lesson/Unit Plan Name:
Fractional Exponents and Property of Exponents

Rationale/Lesson Abstract:
To use the definition of fractional exponents to simplify expressions using property of exponents.

Timeframe:
50 Minutes

Common Core Standard(s):
AlgI.N-RN.1 Explain how the definition of the meaning of rational exponents follows from the extending of the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents.

Activity/Lesson:

Start by taking a look and integer exponents (see the provided handout.) Discuss and remind students of the rules for zero exponents, a negative exponent and properties of exponents.

$$4^3 = 4^{4-1} = \frac{4^4}{4} = \frac{4\cdot4\cdot4\cdot4}{4} = 4\cdot4\cdot4 = 64$$

$$4^2 = \frac{4\cdot4\cdot4}{4} = 4\cdot4 = 16$$

$$4^1 = \frac{4\cdot4}{4} = 4$$

$$4^0 = \frac{4}{4} = 1$$

$$4^{-1} = \frac{4}{4\cdot4} = \frac{1}{4}$$

$$4^{-2} = \frac{4}{4\cdot4\cdot4} = \frac{1}{4\cdot4} = \frac{1}{16}$$

It may be confusing for students to see the exponent of 3 replaced with and equivalent form of $4-1$.

Try explaining it as substitution. Let $3 = 4-1$, so everywhere you see a 3, put $4-1$.

You will also have to explain the converse of the power of a quotient property, which is addressed in the warm up.

$$\frac{4^4}{4} = 4^{4-1}$$

Now let's look at radical expressions.
$\sqrt{4} = 2$, Why? How can you prove this to me or show me how this works?

$\sqrt{2^2} = 2$, find the number times itself.

Again, explain the exponent of 1 is replaced with and equivalent form of $\frac{2}{2}$.

Let $1 = \frac{2}{2}$, so everywhere you see a 1, put $\frac{2}{2}$.

This is what we know: $2 = 2^1 = 2^{\frac{2}{2}} = 2^{2\bullet\frac{1}{2}} = \left(2^2\right)^{\frac{1}{2}} = \left(4\right)^{\frac{1}{2}} = \sqrt{4}$

Therefore, $4^{\frac{1}{2}} = \sqrt{4}$
So, now we know that the square root can be written as a fractional exponent of one half.

Activity/Lesson continued:

Now let's look at a fractional exponent. Where would you put $4^{\frac{1}{2}}$? Between which 2 powers?

Where would you put $4^{\frac{3}{2}}$? Between which 2 powers?

$$4^3 = 4^{4-1} = \frac{4^4}{4} = \frac{4 \cdot 4 \cdot 4 \cdot 4}{4} = 4 \cdot 4 \cdot 4 = 64$$

$$4^2 = \frac{4 \cdot 4 \cdot 4}{4} = 4 \cdot 4 = 16$$

$$4^{\frac{3}{2}} = (4)^{\frac{1}{2}\bullet 3} = (2^2)^{\frac{1}{2}\bullet 3} = \left(2^{2\bullet\frac{1}{2}}\right)^3 = (2^1)^3 = 2^3 = 2 \cdot 2 \cdot 2 = 8$$

$$4^1 = \frac{4 \cdot 4}{4} = 4$$

$$4^{\frac{1}{2}} = (2^2)^{\frac{1}{2}} = 2^{\frac{2}{2}} = 2$$

$$4^0 = \frac{4}{4} = 1$$

$$4^{-1} = \frac{4}{4 \cdot 4} = \frac{1}{4}$$

$$4^{-2} = \frac{4}{4 \cdot 4 \cdot 4} = \frac{1}{4 \cdot 4} = \frac{1}{16}$$

Let's take a look at couple of examples.

1.	2.	3.
$\sqrt{81}$ $= \sqrt{9^2}$ or $= 9$ $\sqrt{81}$ $= (81)^{\frac{1}{2}}$ $= (9^2)^{\frac{1}{2}}$ $= 9^{2\bullet\frac{1}{2}}$ $= 9$	$16^{\frac{1}{2}}$ $= \sqrt{16}$ or $= \sqrt{4^2}$ $= 4$ $16^{\frac{1}{2}}$ $= (4^2)^{\frac{1}{2}}$ $= 4$	$9^{\frac{5}{2}}$ $= 9^{\frac{1}{2}\bullet 5}$ $= \left(9^{\frac{1}{2}}\right)^5$ or $= (\sqrt{9})^5$ $= 3^5$ $= 243$ $9^{\frac{5}{2}}$ $= 9^{\frac{1}{2}\bullet 5}$ $= \left(9^{\frac{1}{2}}\right)^5$ $= \left(3^{2\bullet\frac{1}{2}}\right)^5$ $= 3^5$ $= 243$

You try: $100^{\frac{3}{2}}$

Activity/Lesson continued:

$$100^{\frac{3}{2}} = (100)^{\frac{1}{2}\bullet 3} = \left(100^{\frac{1}{2}}\right)^3 = \left(\sqrt{100}\right)^3 = 10^3 = 1{,}000$$

or

$$100^{\frac{3}{2}} = (100)^{\frac{1}{2}\bullet 3} = \left(100^{\frac{1}{2}}\right)^3 = \left(10^{2\bullet\frac{1}{2}}\right)^3 = 10^3 = 1{,}000$$

You can do this for any fractional exponent or any radical with an indicated index.

$\sqrt[n]{a} = a^{\frac{1}{n}}$, where n is the index.

Let's take a look at a couple of examples.

4.	5.	6.	7.
$\sqrt[3]{a}$ $= a^{\frac{1}{3}}$	$\sqrt[5]{x}$ $= x^{\frac{1}{5}}$	$\sqrt[3]{27}$ $= 27^{\frac{1}{3}}$ $= \left(3^3\right)^{\frac{1}{3}}$ $= 3$ or $\sqrt[3]{27}$ $= \sqrt[3]{3^3}$ $= 3$	$64^{\frac{4}{3}}$ $= \left(64^{\frac{1}{3}}\right)^4$ $= \left(4^{3\bullet\frac{1}{3}}\right)^4$ $= 4^4$ $= 256$ or $64^{\frac{4}{3}}$ $= \left(64^{\frac{1}{3}}\right)^4$ $= \left(\sqrt[3]{64}\right)^4$ $= \left(\sqrt[3]{4^3}\right)^4$ $= 4^4$ $= 256$

You try: $125^{\frac{2}{3}}$

$$125^{\frac{2}{3}} = 125^{\frac{1}{3}\bullet 2} = \left(5^{3\bullet\frac{1}{3}}\right)^2 = 5^2 = 25$$

or

$$125^{\frac{2}{3}} = 125^{\frac{1}{3}\bullet 2} = \left(\sqrt[3]{5^3}\right)^2 = 5^2 = 25$$

Assessment/Homework:

Evaluate the expression using two different methods.

1. $121^{\frac{1}{2}}$

2. $81^{\frac{3}{2}}$

3. $343^{\frac{1}{3}}$

4. $16^{\frac{3}{4}}$

5. $27^{\frac{4}{3}}$

Name:________________________________

Integer and Fractional Exponent Handout

$4^3 =$

$4^2 =$

$4^1 =$

$4^0 =$

$4^{-1} =$

$4^{-2} =$

Warm Up

Common Core	Review	Other
1. Which of the following are equivalent to 4? Select all that apply. A. $\frac{4\cdot 4\cdot 4}{4}$ B. $5-1$ C. $16\cdot\frac{1}{8}$ D. $\sqrt{25}$ E. $\left(2^4\right)^{\frac{1}{2}}$ F. * Change all the expressions not selected to be equivalent to 4.	The **Quotient of Powers Property** states: To divide powers having the same base, subtract exponents. $\frac{a^m}{a^n}=a^{m-n},\quad a\neq 0$ 2. Use the quotient of powers property to simplify the following. $\frac{4^4}{4^1}$ * Write what you do when you use the Quotient of powers property.	3. Use the converse of the quotient of powers property to write the given expression as a quotient of powers. $a^{m-n}=\frac{a^m}{a^n},\quad a\neq 0$ 4^{3-1} * Explain what it means to you to do the converse of the quotient of powers property.